MATHEMATICS HIGHER LEVEL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-14	15-27	28-39	40-51	52-62	63-73	74-100

Portfolio

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-4	5-6	7-9	10-12	13-14	15-17	18-20

General comments

Since this was the second November session requiring internal assessment, it was expected to go more smoothly. I am pleased to report that this was in fact the case. Many of the comments made for the May 2001 report are also relevant for the November session.

The major changes that were made last year in the way in which the internal assessment component of the mathematics HL course was going to be assessed appear to have worked well. Reducing the requirement to only three items for each portfolio has helped candidates, teachers and the moderation team deal with the demands of the internal assessment component.

Though there are still problems with some schools, it seems that the vast majority have put in place a procedure for integrating portfolio work as a teaching tool into the mathematics HL course and that candidates are benefitting from the experience. Using items from the Teacher Support Materials document (TSM) as well as some school written assignments, many schools are creating tasks that require imagination, creativity and insight from their candidates.

However, it should be noted that the number of items selected from the TSM is far greater than teacher developed items. It will have to be seen if a continuation of the use of the same items over several sessions creates additional problems.

Teachers need to correct portfolio work using red pen and where it is appropriate, add additional comments that will be helpful feedback to the candidate. Some items submitted for moderation were devoid of teacher marks or comments, leading one to wonder whether the teacher actually marked the work at all! In the event that the item is part of a sample sent for moderation, these corrections and comments will also be helpful to the moderator. Work that has no evidence of being corrected and has no teacher comments is extremely difficult and time consuming to moderate.

If the items selected by the IBO as part of a school sample are not taken from the TSM, for the moderating process to work well, it is essential that a statement of the task is included along with a set of solutions.

Teachers were assessing some portfolios taken from the TSM as a different type eg a Type I item photocopied from the TSM is given to candidates as a Type II item, or unchanged items being assessed against criteria other than those noted in the TSM. Of course it is possible for teachers to take items from the TSM and adapt them to cover other criteria or types, but if the item is taken as is, then the type and the criteria cannot be changed.

In the calculation of the final mark and the completion of the correct forms (5/IA and 5/PFCS) it is important that teachers follow the instructions carefully. Errors are being made by teachers not fully aware of the correct procedure to follow.

Where more than one instructor is involved, internal assessment criteria should be well defined, agreed upon and upheld.

It should be repeated that it is in the best interests of the candidates if more than three items are completed over the two years. Then the best three can be selected making sure that there is one of each type, and that all the criteria (especially E and F) have been covered. Starting in 2002 there will be a penalty applied to portfolios that are non-compliant.

Specific comments about portfolio work

The vast majority of the work submitted was appropriate. The quality ranged from the superficial to exemplary.

All items in a portfolio should meet the requirements of the mathematics HL course. Portfolios that contained work from the mathematical methods SL TSM are not consistent with the course and therefore could not score as highly, particularly in criterion C that assesses the mathematical content.

All three items should cover a range of topics.

It is clear from work submitted that some candidates probably spent many more than the prescribed number of hours on their portfolios.

Specific comments on assessment criteria

Criteria A Use of notation and terminology

Most candidates are able to score full marks.

Criteria B Communication

For full marks it is important that the candidate makes the paper easy to read and understand without reference to the question. This should include linking words and graphs etc.

Criteria C Mathematical content

This is the area that separates the weaker from the stronger candidates. It is also the one area where many teachers gave marks a little too generously.

Criteria D Results or conclusions

This is an area where candidates should make greater effort. In general more care is needed in giving a clear conclusion, or more detailed analysis of their results or findings.

Criteria E Making conjectures

It is important to select an item that will give the candidates the opportunity to score full marks. Lots of schools used the "Investigating a Sequence of Numbers" from the TSM and there were many examples of excellent work. It would appear that much effort has gone into teaching proof by mathematical induction.

Criteria F Use of technology

More documentation of the use of technology is still needed. Sometimes very little use of technology was awarded 2 or 3 marks. Candidates need more help and guidance on how appropriate technology can be used to explore rather than just calculate results.

Paper 1

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-9	10-19	20-27	28-33	34-40	41-46	47-60

Summary of the G2 forms

Comparison with last year's paper:

much easier	a little easier	of a similar standard	a little more difficult	much more difficult
-	2	11	1	-

Suitability of question paper:

	too easy	appropriate	too difficult
Level of difficulty	-	16	1

	poor	satisfactory	good
Syllabus coverage	-	11	6

	poor	satisfactory	good
Clarity of wording	-	5	12

	poor	satisfactory	good
Presentation of paper	-	4	13

The teacher responses on the G2 form were very favourable.

The assistant examiners were also pleased with the paper but found that candidates had difficulty with the wording of Question 20. The examiners found that the examination allowed the candidates to demonstrate a well-rounded knowledge of the syllabus. However, there was overwhelming evidence that the areas of the syllabus that continue to present problems for the candidates were probability and statistics, counting principles, trigonometry and transformations. Examiners felt that candidates should be encouraged to make better use of their graphical display calculators and to draw diagrams to assist in their understanding of the questions.

General comments about the strengths and weaknesses of the candidates

A summary of the success with which candidates attempted the questions is shown below.

Well done	Reasonable success	Some success	Experienced difficulty
6, 7, 13	2, 3, 5, 9,10, 20	1, 4, 8, 11, 14	12, 15, 16, 17, 18, 19

QUESTION 1: **Statistics**

Answers: (a) $\mathrm{E}(X) = 1200$
(b) $\mathrm{SD}(X) = 20$

Most candidates used the $\mathrm{E}(X) = np$ result to find the mean correctly. However, few were able to successfully calculate the standard deviation of a binomial distribution.

QUESTION 2: **Number and algebra**

Answer: $-\mathrm{i}$

Relatively few candidates were able to transform the quotient of two complex numbers into a complex number by multiplying the numerator and denominator by the complex conjugate of the denominator. Many candidates solved this question by separating z into real and imaginary parts.

QUESTION 3: **Functions and equations**

Answer: $a = -6$

Few candidates used the remainder theorem. Instead they used long division or synthetic division often with the consequence of many numerical mistakes.

QUESTION 4: **Number and algebra**

Answers: (a) $-1.5 < x < 1.5$ or $|x| < \frac{3}{2}$
(b) sum $= 5$

Most candidates failed to use the modulus of r when calculating the values for which the series converges, even though this fact is given in the formula booklet. Most candidates summed the series correctly.

QUESTION 5: **Functions and equations**

Answer: Domain $x \in \mathbf{R}$, $x \neq 2$

This question was answered well by many candidates although algebraic errors were made. Some candidates limited themselves to interchanging x and y. Unfortunately, some candidates still believe that $f^{-1}(x) = \frac{1}{f(x)}$.

QUESTION 6: Matrices and transformations

Answer: $x = 1$ and $y = 8$

Candidates performed well on this question.

QUESTION 7: Calculus

Answer: $a = -4$ and $b = 18$.

This question was well solved by many candidates although some made algebraic errors.

QUESTION 8: Probability

Answer: $\mathrm{E}(X) = 0.441 = \dfrac{2\ln 2}{\pi}$

The most frequent error in this question was integrating the density function instead of $x\,f(x)$. Those who did integrate $x\,f(x)$ often used integration by parts instead of recognising that the integral could be expressed in terms of $\ln(1+x^2)$. Many also have the bad habit of not writing down the limits of integration when writing a definite integral, which is often a source of confusion and error. The most successful method was the use of a graphical display calculator to obtain the integral.

QUESTION 9: Matrices and transformations

Answer: $k = 4$ or $k = 7$

Most candidates realised that they needed to set the determinant equal to 0. But the calculation of the determinant was the source of many algebraic and numerical mistakes. The common mistake was to forget the negative for the second element of the expression in the determinant.

QUESTION 10: Calculus

Answers: (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x - 8\cos x$

(b) $\cos x = \dfrac{1}{2}$

Most candidates were able to obtain the derivative although many were unable to solve the resulting equation in part (b). Many gave the value of the angle rather than the value of the cosine as was asked.

QUESTION 11: Functions and equations

Answer: $1 \le x \le 3$

Methods of solution varied on this question. The best results were for those candidates who used graphs. Others did algebraic manipulations but often they did not know whether to take the union or intersection of the solution sets.

QUESTION 12: **Matrices and transformations**

Answer: $\begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{2} & 1 \\ \frac{\sqrt{3}}{2} & 0 \end{pmatrix}$

Most candidates were unable to solve this question. Many did not even attempt it while others treated the transformation as a rotation. Even those who correctly found the coordinates of the top vertex of the transformed triangle were unable to convert this into the correct transformation matrix.

QUESTION 13: **Calculus**

Answers:
(a) The equation of the tangent is $y = -4x - 8$
(b) The point where the tangent meets the curve again is $(-2, 0)$

This question was solved well by most candidates. A few candidates stated that the tangent could not intersect the curve again because it was a tangent.

QUESTION 14: **Vector geometry**

Answer: The minimum distance is 1.63 (3 s.f.)

The majority of candidates were unable even to start this question correctly. Even those who found AP as a function of x failed to minimise it. Many candidates tried to find the equation of the normal to the curve from point A, find the coordinates of P and then apply the distance formula. Few candidates found the right solution. Many forgot to take the square root at the end.

QUESTION 15: **Vector geometry**

Answer: $(7, -4, 0)$ or $(-1, 4, -4)$

Relatively few candidates solved this question correctly. Many gave points that were not even on the line. A common mistake was to find the modulus of the vector $(3+2\lambda, -2\lambda, -2+\lambda)$.

QUESTION 16: **Vector geometry**

Answer: $2\sin\frac{1}{2}\theta$

This question turned out to be one of the most difficult for the candidates. Most were unable to even start the question. Those who did often went into incredibly complicated and inconclusive procedures.

QUESTION 17: **Number and algebra**

Answer: 3168

There were incredibly poor answers to this question. Many candidates tried to count the solutions. Others used incorrect combinatorial arguments.

QUESTION 18: **Probability**

Answer: $\frac{2}{5}$

Most candidates drew a tree diagram but failed to realise that the umbrella could not be left in shop 1 and in shop 2. The answer frequently given was 2/9 because the candidates did not realise that they were asked for a conditional probability.

QUESTION 19: **Functions and equations**

Answer: half-life = 170 years (3 s.f.)

In this question many candidates failed to see that the question involved a differential equation. Many assumed that the decay rate was constant and so used proportionality instead.

QUESTION 20: **Calculus**

Answer: Area $= 1.22$

This question was fairly well answered although many candidates misunderstood the question. The mention of $-3 \le x \le 3$ led many to think that –3 and 3 were the two limits of integration. Many continued correctly with this assumption and found the area between the curves to be 4.50. Other candidates, however, assumed incorrectly that one could simply integrate from –3 to 3.

Paper 2

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0-12	13-24	25-34	35-45	46-56	57-67	68-100

Summary of the G2 forms

Comparison with last year's paper:

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Clarity of wording	-	5	12

	poor	satisfactory	good
Presentation of paper	-	4	13

General comments

- The graph plotting, particularly in Question 4, was generally poor. Candidates need to ensure that the choice of window in their calculator allows all the branches of the graph to be visible.
- Probability continues to be an area of difficulty with even the simpler parts of problems inaccessible to many candidates.
- As usual, the induction question caused problems for many candidates. There are still many candidates who seem unaware that the essential step is to assume that the proposition is true for $n = k$ and then show that this implies that the proposition is true for $n = k + 1$.
- Many candidates are losing a mark through an LAP (level of accuracy penalty) although, luckily for them, only one such mark can now be lost on a paper.
- Many candidates seem to be unfamiliar with complex number calculations, especially those involving a switch from Cartesian to polar form and vice versa.
- In those questions which could be solved either by calculator or analytically, e.g. definite integration or location of maximum/minimum points, those candidates who use their calculator are generally more successful than those who try to use theoretical methods. Teachers should be aware of this, and should perhaps recommend their candidates to use their calculators in these situations.
- As usual, the Section B questions were very poorly answered. Some of the attempted solutions were so poor that one wondered if some centres had covered the optional material even superficially. The question on Sets, Relations and Groups was answered the best and the question on Geometry the worst.

Section A

QUESTION 1: **Calculus**

Answers :

(a) $x = 1,\ y = -1,\ z = 2$

(b) $\boldsymbol{v} = 11\boldsymbol{i} - 7\boldsymbol{j} - 5\boldsymbol{k}$

(d) Thus, an equation for l is $\boldsymbol{r} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k} + \lambda(6\boldsymbol{i} + 13\boldsymbol{j} - 5\boldsymbol{k})$, where λ is a scalar.

Most candidates were able to solve part (a) correctly using a variety of methods, i.e. straightforward elimination, reduction to echelon form, use of an inverse matrix and straightforward use of a calculator. Most candidates solved part(b) correctly although arithmetic errors were seen. In part (c), some candidates simply showed that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{a}$ and $\boldsymbol{b}$ and then failed to demonstrate that this implied that $\boldsymbol{u}$ is perpendicular to any linear combination of $\boldsymbol{a}$ and $\boldsymbol{b}$. Part (d) proved to be very difficult with many candidates apparently unaware of the difference between the equation of a line and the equation of a plane.

QUESTION 2: Functions and equations

Answers:

(ii) (a) $t = 0,\ t = 3\ or\ t = 6$

(b) (i) The required distance,

$$d = \int_0^3 t\sin\left(\frac{\pi}{3}t\right)\mathrm{d}t - \int_3^6 t\sin\left(\frac{\pi}{3}t\right)\mathrm{d}t$$

(ii) 11.5 m

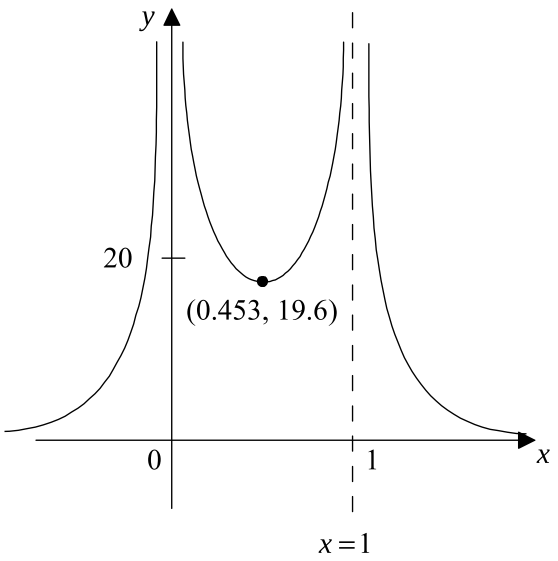
Most candidates solved part (i) correctly although some candidates failed to use the product rule. Solutions to part (ii) were often disappointing. In part (a), at least one of the zeroes was often missed. In part (b), many candidates failed to spot the sign change in v and simply integrated from 0 to 6. Candidates who used their calculator to perform the integration were generally more successful than those who tried to use integration by parts, which usually proved too difficult algebraically.

QUESTION 3: Calculus

Answers:

(a) Therefore, $\mathrm{A} = \left(-\frac{1}{m}, 0\right)$ and $\mathrm{B} = \left(\frac{1}{1-m}, \frac{1}{1-m}\right)$

(c) The graph of $y = \frac{x^2+1}{x^2(1-x)^2}$ is as follows:



(d) From part (c), l is a minimum when $m = 0.453$ as $0 < m < 1$ and then the minimum value of $l = 4.43\left(\sqrt{19.63}\right)$

Most candidates solved part (a) and then part (b) correctly. In part (c), the majority of candidates failed to include the central part of the graph, presumably through using an incorrect range for y. Many candidates therefore failed to locate the minimum point so that part (d) became inaccessible. Of those candidates who did attempt to locate the minimum point, those using their calculator were the most successful. Many failed to indicate the asymptotes as required. Some candidates failed to spot the link between parts (c) and (d).

QUESTION 4: Number and algebra

Answers: (ii) (a) $z^5 - 1 = (z-1)(z^4 + z^3 + z^2 + z + 1)$

(b) $z = 1,$

$$\cos\left(\pm\frac{2\pi}{5}\right) + i\sin\left(\pm\frac{2\pi}{5}\right), \text{or} \cos\left(\pm\frac{4\pi}{5}\right) \pm i\sin\left(\pm\frac{4\pi}{5}\right)$$

(c) $z^4 + z^3 + z^2 + z + 1 = \left(z^2 - \left[2\cos\frac{2\pi}{5}\right]z + 1\right)\left(z^2 + \left[2\cos\frac{\pi}{5}\right]z + 1\right)$

In part (i), many candidates tried to show that the proposition is true for $n = 1$ instead of $n = 2$. Many candidates realised that they had to show that 'true for $n = k \Rightarrow$ true for $n = k + 1$' but some were unable to do this in this context. There are still, however, many candidates who appear not to understand the basis of a proof by induction. In part (ii), although some candidates were able to factorise $z^5 - 1$, few were able to find the five fifth roots of unity. Very few correct solutions to part (c) were seen.

QUESTION 5: Probability

Answers: (a) (i) $\frac{5}{36}$

(ii) $\frac{25}{216}$

(iii) $\left(\frac{5}{6}\right)^{2(n-1)} \times \frac{1}{6}$

(c) $\text{P(Bridget wins)} = 1 - p = \frac{5}{11}$

(d) 0.432

This question was, in general, poorly answered. Although some candidates managed to solve part (a) correctly, few candidates were able to produce the logical argument required to solve part (b). Some enterprising candidates solved part (c) correctly using the result given in part (b) even though they were unable to obtain that result. Few candidates realised that the binomial distribution was needed to solve part (d).

Section B

QUESTION 6: **Statistics**

Answers:

(i) (a) $P(W > 12) = 0.841$

(b) (i) $P\left(\sum W < 65\right) = 0.0680$

(ii) 0.0312

(iii) $P(\bar{W} > 13) = 0.932$

(ii) (a) 95 % C.I. for average reduction is $(0.469, 2.63)$

(b) Reject H_0 and conclude that the new policy does result in a reduction in the number of passengers.

(iii) We conclude that there seems to be no association between the day of production and the quality of the part.

Solutions to part (i) were disappointing with many errors made. In part (ii), few candidates knew how to construct a confidence interval correctly and hardly any candidates were able to carry out the required test in part (b). Some candidates thought that the confidence interval found in part (a) could be used to solve part (b), failing to realise that the confidence interval is two-sided and the required test one-sided. In part (iii), most candidates failed to combine the last two rows.

QUESTION 7: **Sets, relations and groups**

Answer: (ii) (b) The three classes are $\{\{a, c, e\}, \{b, d\}, \{f\}\}$

In part (i)(a), candidates often failed to show that the identity and inverse actually belonged to the set. In part (i)(b), few candidates dealt satisfactorily with the 'if and only if'. In part (ii), candidates often 'proved' reflexivity by noting that *aRa* but failed to mention any other elements – similarly for symmetry. In part (iii), attempts at part (a) were generally poor with candidates often assuming commutativity from the outset. A few correct solutions to part (b) were seen but part (c) defeated almost all the candidates.

QUESTION 8: **Discrete mathematics**

Answers:

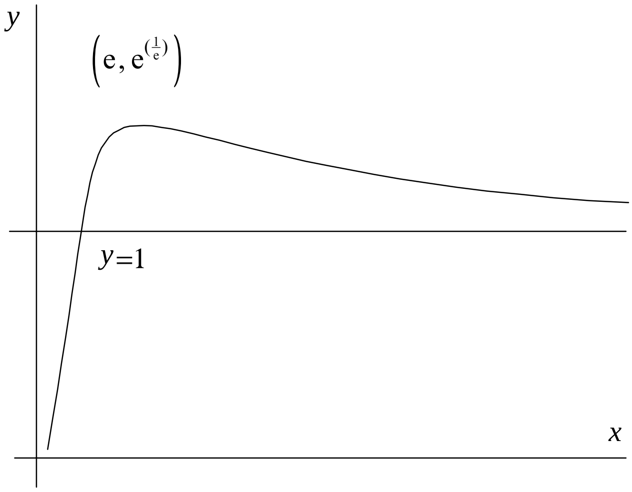
(ii) We conclude that the shortest path is A, B, C, D, E, F and has length 13.

(iii) (d) $a_n = 3^n$

In part (i), it was often not clear what algorithm was being used. Candidates should be encouraged to explain their method so that method marks can be awarded if errors are made. Part (ii) required Dijkstra's Algorithm to be used and it was often not clear from the scripts that this was being done. Candidates who simply wrote down, by inspection, that the shortest path was ABCDEF were not given full credit. In part (iii), candidates often missed the point in parts (a), (b) and (c) but many candidates solved part (d) correctly.

QUESTION 9: Analysis and approximation

Answers: (i) (b)



(c) By Taylor's theorem we have

$$P_2(x) = f(\mathrm{e}) + f'(\mathrm{e})(x-\mathrm{e}) + \frac{f''(\mathrm{e})}{2}(x-\mathrm{e})^2$$

(ii) a_n diverges.

(iv) (b) $n = 75$

Solutions to part (i) were usually disappointing. In part (a), few candidates realised that taking logs would enable the differentiation to be carried out. In part (b), most of the successful candidates used their calculator to locate the maximum and the asymptotes. Hardly any correct solutions were seen to part (c). Few candidates knew how to establish the convergence or otherwise of the series in part (ii). Some candidates suggested that a comparison with the harmonic series might be helpful but were unable to fill in the mathematical details. Solutions to part (iii) were usually disappointing with only a few correct solutions seen. Part (iv) proved to be too difficult for almost all of the candidates.

QUESTION 10: Euclidean geometry and conic sections

Solutions to this question were generally extremely poor. Although some candidates made progress in parts (a) and (b), few candidates made any progress whatsoever in the remaining parts of the question. Most candidates showed no evidence at all of understanding the term 'harmonic division' and very few candidates seemed able to solve problems involving the nine-point circle.

